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Studies on Distillation VIII: The Effect of Carryover on Plate Efficiency*

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Summary

General balance relations are introduced for plate columns with carryover of each phase in both directions. It is shown that these relations can be formally reduced to equivalent standard forms without carryover terms by suitably defining reduced flow rates and concentrations (enthalpies). Using the latter variables, it is then possible to define generalized reduced Murphree and Hausen vapor and liquid plate efficiencies which are mutually consistent and satisfy the basic relations. Both carryovers and these efficiencies can be found from complete concentration measurements, or, inversely, concentrations can be calculated from plate to plate, knowing the carryovers and plate efficiencies.

As is discussed in our survey article (*1*), the question of the influence of liquid carryover on the separating ability of distillation columns and specifically on the efficiency of their trays has not yet been satisfactorily analyzed, despite its obvious importance in diminishing the separation achieved. The relative disregard of this question or, more precisely, the nearly complete lack of a consistent simultaneous analysis of carryover and plate efficiency is the more surprising, as it has long been widely understood that liquid carryover is one of the main factors limiting the capacity of distillation equipment. It may be pointed out that this carryover constitutes a form of axial mixing along

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the column, and that the influence of such mixing in continuous separation equipment such as packed towers has also been neglected until recently, although here also the separation is significantly decreased by this mixing, even under normal operating conditions. Indeed, it is not too much to say that as both capacity and rate of mass transfer increase with throughput, it is desirable to operate separation equipment at as high a loading as possible, and that the optimum loading is frequently determined by the permissible carryover or axial mixing along the column.

These considerations are not limited to distillation columns but apply to all forms of multiphase separation equipment, such as absorption, extraction, and leaching plate columns, for example. Especially with the latter, we may have carryover of each phase by the other along the column. In addition, we may also have weeping, dumping, or partial short-circuiting of liquid through a distillation tray, so that to have full generality we must

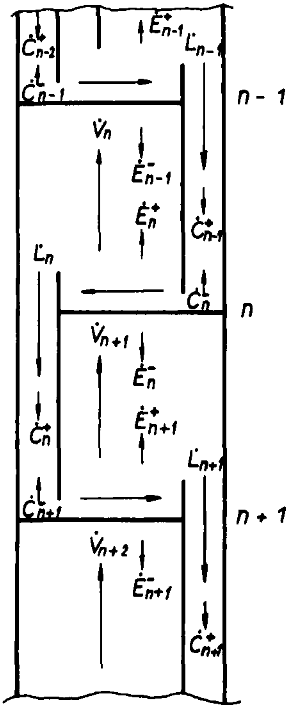

FIG. 1. Section of plate column with carryover streams.

consider "carryover" or entrainment of each phase by the other in both directions from a given tray in addition to the main flows, although we rarely encounter more than two such carryovers occurring simultaneously and frequently only one under given operating conditions. We shall use the terminology of distillation, but the results apply to any stagewise separation equipment if we substitute light phase for vapor and heavy phase for liquid. A section of such a column is shown in Fig. 1, where we have shown a column with cross-flow trays with downcomers for definiteness, but the considerations are not limited to this case.

The over-all material balance above the *n*th tray in the feed-to-condenser section[*] thus takes the form, for steady-state conditions,

$$\dot{V}_n + \dot{E}_n^+ + \dot{C}_n^- = \dot{L}_{n-1} + \dot{C}_{n-1}^+ + \dot{E}_{n-1}^- + \dot{D} \tag{1}$$

where $\dot{V}$ and $\dot{L}$ are the actual (i.e., total) main vapor and liquid flow rates, $\dot{E}^+$ and $\dot{E}^-$ the liquid (upward) carryover and the (downward) weep flow rates, $\dot{C}^+$ and $\dot{C}^-$ the vapor (downward) carryover and (upward) leakage flow rates, and $\dot{D}$ the distillate flow rate. [It should be noted that we can have "forward" carryover ($\dot{E}^-$ and $\dot{C}^-$) in the direction of the main flow of the phase only if the main streams are separated between stages, i.e., if we have downcomers or their equivalent. However, "backward" carryovers ($\dot{E}^+$ and $\dot{C}^+$), in the direction of the main flow of the other phase, are of general occurrence and importance.]

Similarly, for any constituent of a mixture we have the balance relation

$$\bar{y}_n\dot{V}_n + x_n^+\dot{E}_n^+ + y_n^-\dot{C}_n^- = \bar{x}_{n-1}\dot{L}_{n-1} + y_{n-1}^+\dot{C}_{n-1}^+ + x_{n-1}^-\dot{E}_{n-1}^- + x_D\dot{D} \tag{2}$$

where $\bar{y}$ and $\bar{x}$ are the mean concentrations of the main streams and where the carryover concentrations are intermediate values determined by the degree of mixing of the phases on the trays and by the distribution of the carryover across them. We may take

$$\begin{aligned} x_n^+ &\equiv \epsilon_n^+\bar{x}_n + (1-\epsilon_n^+)\bar{x}_{n-1} \\ x_{n-1}^- &\equiv \epsilon_{n-1}^-\bar{x}_{n-1} + (1-\epsilon_{n-1}^-)\bar{x}_{n-2} \\ y_{n-1}^+ &\equiv \gamma_{n-1}^+\bar{y}_{n-1} + (1-\gamma_{n-1}^+)\bar{y}_n \\ y_n^- &\equiv \gamma_n^-\bar{y}_n + (1-\gamma_n^-)\bar{y}_{n+1} \end{aligned} \tag{3}$$

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[*] The analysis below the feed is very similar; simply replace $\dot{D}$ by $-\dot{W}$, x_D by x_W, etc. (2).

where the parameters ϵ^+, ϵ^-, γ^+, and γ^-, which we expect to lie in the range <0,1>, express the degree of mixing of the phase and the distribution of the "carryover" on the plate. (Strictly speaking, we should consider different ϵ's and γ's for each constituent, but the differences are certainly very small and may be neglected in practice.) Thus if we take the effective flow rates

$$\begin{aligned}
\dot{V}_n'' &\equiv \dot{V}_n + \gamma_n^- \dot{C}_n^- - (1 - \gamma_{n-1}^+)\dot{C}_{n-1}^+ \\
\dot{E}_n^{+\prime\prime} &\equiv \epsilon_n^+ \dot{E}_n^+ \\
\dot{C}_n^{-\prime\prime} &\equiv (1 - \gamma_n^-)\dot{C}_n^- \\
\dot{L}_{n-1}'' &\equiv \dot{L}_{n-1} + \epsilon_{n-1}^- \dot{E}_{n-1}^- - (1 - \epsilon_n^+)\dot{E}_n^+ \\
\dot{C}_{n-1}^{+\prime\prime} &\equiv \gamma_{n-1}^+ \dot{C}_{n-1}^+ \\
\dot{E}_{n-1}^{-\prime\prime} &\equiv (1 - \epsilon_{n-1}^-)\dot{E}_{n-1}^-
\end{aligned} \tag{4}$$

we can write the balance relations in terms of the average stream concentrations

$$\dot{V}_n'' + \dot{E}_n^{+\prime\prime} + \dot{C}_n^{-\prime\prime} = \dot{L}_{n-1}'' + \dot{C}_{n-1}^{+\prime\prime} + \dot{E}_{n-1}^{-\prime\prime} + \dot{D} \tag{5}$$

and

$$\begin{aligned}
&\bar{y}_n \dot{V}_n'' + \bar{x}_n \dot{E}_n^{+\prime\prime} + \bar{y}_{n+1} \dot{C}_n^{-\prime\prime} \\
&\qquad = \bar{x}_{n-1} \dot{L}_{n-1}'' + \bar{y}_{n-1} \dot{C}_{n-1}^{+\prime\prime} + \bar{x}_{n-2} \dot{E}_{n-1}^{-\prime\prime} + x_D \dot{D}
\end{aligned} \tag{6}$$

These equations may be considered the working form of the material balance for a plate with backward and forward carryovers in both streams.

We choose in our further development to define certain reduced flow rates and concentrations which satisfy balance relations of the usual form without carryovers. We take the reduced flow rates: for vapor,

$$\dot{V}_n' \equiv \dot{V}_n'' + \dot{C}_n^{-\prime\prime} - \dot{C}_{n-1}^{+\prime\prime} = \dot{V}_n + \dot{C}_n^- - \dot{C}_{n-1}^+ \tag{7a}$$

and for liquid,

$$\dot{L}_{n-1}' \equiv \dot{L}_{n-1}'' + \dot{E}_{n-1}^{-\prime\prime} - \dot{E}_n^{+\prime\prime} = \dot{L}_{n-1} + \dot{E}_{n-1}^- - \dot{E}_n^+ \tag{7b}$$

so the over-all material balance becomes

$$\dot{V}_n' = \dot{L}_{n-1}' + \dot{D} \tag{8}$$

and then define the corresponding reduced concentrations as

$$\begin{aligned} \bar{y}'_n\dot{V}'_n &\equiv \bar{y}_n\dot{V}''_n + \bar{y}_{n+1}\dot{C}_n^{-''} - \bar{y}_{n-1}\dot{C}_{n-1}^{+''} \\ \bar{x}'_{n-1}\dot{L}'_{n-1} &\equiv \bar{x}_{n-1}\dot{L}''_{n-1} + \bar{x}_{n-2}\dot{E}_{n-1}^{-''} - \bar{x}_n\dot{E}_n^{+''} \end{aligned} \tag{9}$$

so the constituent material balance becomes

$$\bar{y}'_n\dot{V}'_n = \bar{x}'_{n-1}\dot{L}'_{n-1} + x_D\dot{D} \tag{10}$$

It is important to realize that Eqs. (8) and (10) represent only a formal simplification of the problem of solving the material balances (5) and (6) and that they apply to a real plate with carryovers. At most we can further say that they could also apply to a column without carryovers with the indicated reduced flow rates and concentrations, assuming it could operate under these conditions. We cannot, however, claim that these are the material balances of an actual column operating without carryovers, and they in no sense represent a solution of Eqs. (5) and (6); this can be seen from the fact that Eqs. (8) and (10) define a reduced operating line which will be the same for columns operating with given reduced flow rates irrespective of the actual carryovers in the columns. Mathematically stated, Eqs. (9), if taken literally, would mean neglecting forward and backward first differences of $\bar{y}_n$ and $\bar{x}_{n-1}$ in the solution of the problem. Nevertheless, these relations are useful for analyzing the operation of the real column, particularly as they suggest suitable methods of comparison.

It may be pointed out that there are a number of other possible choices for defining reduced (or augmented) streams and compositions. For example, we could define augmented streams and compositions by adding the carryover streams to the streams carrying them, but this procedure would involve combining streams of differing states and hence seems less suitable physically (see Ref. *1*).

Returning to our choice, it is convenient to consider the important special case where only liquid carryover is important (i.e., $\dot{E}^+ > 0$, $\dot{E}^- = \dot{C}^+ = \dot{C}^- = 0$), when

$$\dot{V}'_n = \dot{V}''_n = \dot{V}_n \qquad \text{and} \qquad \dot{L}'_{n-1} = \dot{L}''_{n-1} - \dot{E}_n^{+''} = \dot{L}_{n-1} - \dot{E}_n^{+} \tag{11}$$

so that

$$\bar{y}'_n = \bar{y}_n \qquad \text{and} \qquad \bar{x}'_{n-1}\dot{L}'_{n-1} = \bar{x}_{n-1}\dot{L}'_{n-1} + (\bar{x}_{n-1} - \bar{x}_n)\dot{E}_n^{+''} \tag{12}$$

and since for separation $\bar{x}_{n-1} \geqslant \bar{x}_n$,

$$\bar{x}'_{n-1} \geqslant \bar{x}_{n-1} \tag{13}$$

Alternatively, we can now write

$$\bar{y}_n = \bar{x}_{n-1}(\dot{L}'_{n-1}/\dot{V}'_n) + (\bar{x}_{n-1} - \bar{x}_n)(\dot{E}_n^{+''}/\dot{V}'_n) + x_D(\dot{D}/\dot{V}'_n) \tag{14}$$

which is the unreduced or "true" operating line equation for the upper part of the column with reduced flow rates. The corresponding reduced operating line equation without holdup is

$$\bar{y}'_n = \bar{x}'_{n-1}(\dot{L}'_{n-1}/\dot{V}'_n) + x_D(\dot{D}/\dot{V}'_n) \tag{15}$$

These relations may be shown graphically for binaries on a $y - x$ diagram (see Fig. 2). Whereas the reduced operating line will be straight if the reduced reflux ratio

$$R'_n \equiv \dot{L}'_{n-1}/\dot{V}'_n \tag{16}$$

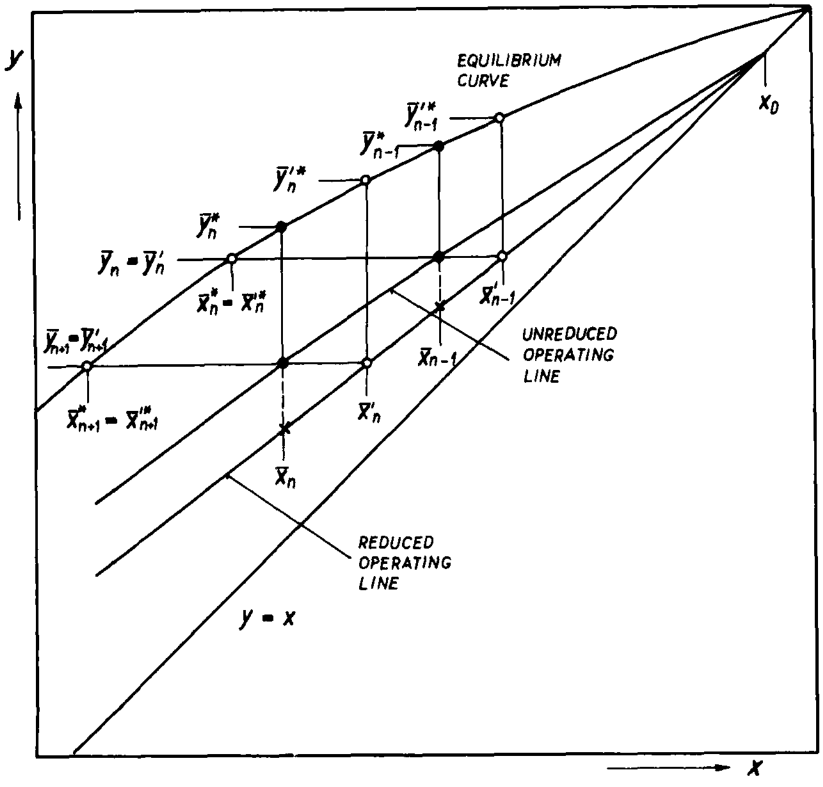

FIG. 2. Operating lines with liquid carryover.

is constant from plate to plate, the operating line (14) will in general be curved, even when this ratio and the relative liquid carryover

$$e_n^{+\prime} \equiv \dot{E}_n^{+\prime\prime}/\dot{V}_n' \tag{17}$$

is constant from plate to plate.

It is important to note that Eq. (14) can also be written

$$e_n^{+\prime} = \left(\frac{\bar{y}_n - \bar{x}_{n-1}}{\bar{x}_{n-1} - \bar{x}_n}\right) - \left(\frac{x_D - \bar{x}_{n-1}}{\bar{x}_{n-1} - \bar{x}_n}\right)(1 - R_n') \tag{18}$$

We can thus determine the liquid carryover from a knowledge of the actual stream concentrations around a plate and the reduced reflux ratio. Note that if this ratio is constant from plate to plate in the rectifying section, and if there is no liquid carryover from the top plate into the condenser, R_n' is also the actual external reflux ratio, R_0. In particular for total reflux ($R_n' = 1$), the relative liquid carryover is given by the first term in Eq. (18).

Another case of importance, with, for example, sieve trays at low vapor loadings, is that with only liquid weeping (i.e., $\dot{E}^- > 0$, $\dot{E}^+ = \dot{C}^+ = \dot{C}^- = 0$), when again

$$\dot{V}_n' = \dot{V}_n'' = \dot{V}_n \qquad \text{but now} \qquad \dot{L}_{n-1}' = \dot{L}_{n-1}'' + \dot{E}_{n-1}^{-\prime\prime} = \dot{L}_{n-1} + \dot{E}_{n-1}^- \tag{19}$$

so that again

$$\bar{y}_n' = \bar{y}_n \qquad \text{but now} \qquad \bar{x}_{n-1}'\dot{L}_{n-1}' = \bar{x}_{n-1}\dot{L}_{n-1}' + (\bar{x}_{n-2} - \bar{x}_{n-1})\dot{E}_{n-1}^{-\prime\prime} \tag{20}$$

and since for separation $\bar{x}_{n-2} \geqslant \bar{x}_{n-1}$, again

$$\bar{x}_{n-1}' \geqslant \bar{x}_{n-1} \tag{21}$$

Thus here also the unreduced operating line lies above and to the left of the reduced one. In both cases the actual separation is more difficult (i.e., the operating line is closer to the equilibrium curve) than would be the case for a column operating with the reduced parameters without carryovers, which conclusion is of course true in general.

Just as many choices of reduced operating conditions are possible, so are there many possible ways of defining plate efficiencies with carryovers (*1*). The basic problem here is the question of the definition of the equilibrium conditions on a theoretical plate. This definition should be consistent with the material balance conditions, reduce to the classical forms for plates without carryovers, and give plate efficiencies which satisfy the basic relations between these efficiencies [see Eqs. (24) and (30)]. The only choice which is physically plausible and which satisfies all these conditions is the one using the reduced variables, which is the reason we employ it.

Thus for the Murphree plate efficiencies we take (2,3)

$$\bar{E}'_{MV} \equiv \frac{\dot{V}'_n \bar{y}'_n - \dot{V}'_{n+1}\bar{y}'_{n+1}}{\dot{V}'^{*}_n \bar{y}'^{*}_n - \dot{V}'_{n+1}\bar{y}'_{n+1}} \tag{22}$$

for the vapor and

$$\bar{E}'_{ML} \equiv \frac{\dot{L}'_{n-1}\bar{x}'_{n-1} - \dot{L}'_n \bar{x}'_n}{\dot{L}'_{n-1}\bar{x}'_{n-1} - \dot{L}'^{*}_n \bar{x}'^{*}_n} \tag{23}$$

for the liquid for each constituent, where $\bar{y}'^{*}_n$ is the vapor concentration in equilibrium with the liquid concentration $\bar{x}'_n$, and, similarly, $\bar{x}'^{*}_n$ is in equilibrium with $\bar{y}'_n$. The Murphree plate efficiencies are applied in situations where we can assume constant (reduced) flow rates from plate to plate on the actual as well as the reference theoretical plates, when these relations simplify by canceling the flow rates, giving the more usual forms. In this case, if we can further assume a linear equilibrium relation over the range of concentrations occurring on the plate, we find the well-known binary relation

$$\left(\frac{\bar{E}'_{ML}}{1-\bar{E}'_{ML}}\right) = \frac{m\dot{V}'}{\dot{L}'}\left(\frac{\bar{E}'_{MV}}{1-\bar{E}'_{MV}}\right) \tag{24}$$

where m is the slope of the equilibrium line.

Although we can also define other "real" Murphree efficiencies using, for example, the actual flow rates and concentrations, they do not satisfy this relation and in general are unsatisfactory.

If we consider the case with only liquid carryover (or weep) [cf. Eqs. (13) and (21)], we see that we obtain the Murphree vapor efficiencies by projecting the actual vapor concentrations horizontally to the reduced operating line and reading off the vertical distances $\bar{y}'_n - \bar{y}'_{n+1}$ and $\bar{y}'^{*}_n - \bar{y}'_{n+1}$. We see in fact that to find these efficiencies we do not need to know the values of the liquid concentrations at all, if we know the position of the reduced operating line. This procedure is in exact contradiction to the usual and incorrect procedure where only liquid concentrations are measured, the possibility of carryover not taken into account and the reduced operating line employed to calculate a vapor efficiency essentially by projecting these liquid concentrations vertically to this reduced operating line to give "reduced" points, which are then used in the usual manner to calculate the Murphree vapor efficiency. The fact that the actual vapor and liquid concentrations do not lie on the reduced operating line is overlooked. It is necessary to note that even if liquid Murphree plate efficiencies were calculated from these vertically projected points, the resulting values would also not be consistent with the material balances. It is also pointless to try to correct such inconsistent efficiencies a posteriori by analyses of the effect of carryover on the operation of the plate. It is important to emphasize that virtually all the Murphree plate efficiencies (usually vapor) reported in the literature (*3*) were obtained in this unjustified manner and that carryover is not at all negligible in its effect on plate concentrations and efficiencies for heavy plate loadings representative of industrial practice.

It must also be emphasized that the Murphree plate efficiencies defined by Eqs. (22) and (23) are also "real" (more exactly "real reduced") efficiencies and that they refer to the conditions in the actual column. As we have discussed elsewhere (*1*), we do not see how "carryover-free" efficiencies can be obtained from data on a column operating with carryover, but that by a proper interpretation of the data from the real column we can find, for example, the actual mass-transfer coefficients while taking account of carryover as well as liquid mixing on the plate, as will be developed elsewhere.

In another paper (*2*) we have criticized the Murphree plate efficiencies from a number of points of view and our criticisms apply equally well to these reduced efficiencies; we have discussed them at length, as so much of the data in the literature is reported in terms of Murphree efficiencies. In their place we can use our generalization of the Hausen efficiency (*4*), again employing reduced conditions. We supplement the material balances (5) and (6) with the enthalpy balance, assuming an adiabatic column, for simplicity:

$$\begin{aligned}\bar{H}_n\dot{V}_n'' + \bar{h}_n\dot{E}_n^{+''} + \bar{H}_{n+1}\dot{C}_n^{-''} = \bar{h}_{n-1}\dot{L}_{n-1}'' + \bar{H}_{n-1}\dot{C}_{n-1}^{+''} \\ + \bar{h}_{n-2}\dot{E}_{n-1}^{-''} + h_D\dot{D} + \dot{Q}_D \quad (25)\end{aligned}$$

where $\bar{H}$ and $\bar{h}$ are the actual vapor and liquid molal enthalpies and $\dot{Q}_D$ is the condenser heat duty. Again we introduce the reduced flow rates (7) and the corresponding reduced molal enthalpies

$$\begin{aligned} \bar{H}'_n\dot{V}'_n &\equiv \bar{H}_n\dot{V}''_n + \bar{H}_{n+1}\dot{C}_n^{-\prime\prime} - \bar{H}_{n-1}\dot{C}_{n-1}^{+\prime\prime} \\ \bar{h}'_{n-1}\dot{L}'_{n-1} &\equiv \bar{h}_{n-1}\dot{L}''_{n-1} + \bar{h}_{n-2}\dot{E}_{n-1}^{-\prime\prime} - \bar{h}_n\dot{E}_n^{+\prime\prime} \end{aligned} \tag{26}$$

when the enthalpy balance becomes

$$\bar{H}'_n\dot{V}'_n = \bar{h}'_{n-1}\dot{L}'_{n-1} + h_D\dot{D} + \dot{Q}_D \tag{27}$$

Now we may define the equilibrium state for a theoretical plate by the relations

$$\begin{aligned} \dot{V}'_{n+1} + \dot{L}'_{n-1} &= \dot{V}'_n + \dot{L}'_n \equiv \dot{V}'^{*}_n + \dot{L}'^{*}_n \\ \bar{y}'_{n+1}\dot{V}'_{n+1} + \bar{x}'_{n-1}\dot{L}'_{n-1} &= \bar{y}'_n\dot{V}'_n + \bar{x}'_n\dot{L}'_n \equiv \bar{y}'^{*}_n\dot{V}'^{*}_n + \bar{x}'^{*}_n\dot{L}'^{*}_n \\ \bar{H}'_{n+1}\dot{V}'_{n+1} + \bar{h}'_{n-1}\dot{L}'_{n-1} &= \bar{H}'_n\dot{V}'_n + \bar{h}'_n\dot{L}'_n \equiv \bar{H}'^{*}_n\dot{V}'^{*}_n + \bar{h}'^{*}_n\dot{L}'^{*}_n \end{aligned} \tag{28}$$

obtained from Eqs. (8), (10), and (27) by differencing. We define our new ("real reduced") over-all plate efficiencies by the relations

$$\bar{E}'_V \equiv \frac{\dot{V}'_n - \dot{V}'_{n+1}}{\dot{V}'^{*}_n - \dot{V}'_{n+1}} \qquad \bar{E}'_L \equiv \frac{\dot{L}'_{n-1} - \dot{L}'_n}{\dot{L}'_{n-1} - \dot{L}'^{*}_n} \tag{29}$$

but we see immediately that

$$\bar{E}'_V = \bar{E}'_L = \bar{E}' \tag{30}$$

Similarly, for each constituent

$$\bar{E}'_{Hy} \equiv \frac{\dot{V}'_n\bar{y}'_n - \dot{V}'_{n+1}\bar{y}'_{n+1}}{\dot{V}'^{*}_n\bar{y}'^{*}_n - \dot{V}'_{n+1}\bar{y}'_{n+1}} \qquad \bar{E}'_{Hx} \equiv \frac{\dot{L}'_{n-1}\bar{x}'_{n-1} - \dot{L}'_n\bar{x}'_n}{\dot{L}'_{n-1}\bar{x}'_{n-1} - \dot{L}'^{*}_n\bar{x}'^{*}_n} \tag{29'}$$

formally identical with the Murphree plate efficiencies (22) and (23), but with the equilibrium state (28), so (3,5)

$$\bar{E}'_{Hy} = \bar{E}'_{Hx} = \bar{E}'_{Hi} \tag{30'}$$

Finally, we define the thermal efficiencies

$$\bar{E}'_{HH} \equiv \frac{\dot{V}'_n\bar{H}'_n - \dot{V}'_{n+1}\bar{H}'_{n+1}}{\dot{V}'^{*}_n\bar{H}'^{*}_n - \dot{V}'_{n+1}\bar{H}'_{n+1}} \qquad \bar{E}'_{Hh} \equiv \frac{\dot{L}'_{n-1}\bar{h}'_{n-1} - \dot{L}'_n\bar{h}'_n}{\dot{L}'_{n-1}\bar{h}'_{n-1} - \dot{L}'^{*}_n\bar{h}'^{*}_n} \tag{29''}$$

and again

$$\bar{E}'_{HH} = \bar{E}'_{Hh} = \bar{E}'_H \tag{30''}$$

as in the case without carryovers (2).

Thus far we have considered the case where from experimental data we wish to calculate the carryovers and the plate efficiencies. In the inverse problem, where we know the carryovers and efficiencies, we wish to calculate the concentrations from plate to plate along the column. For brevity we shall consider the case with only constant liquid carryover, constant molal flow rates, and Murphree vapor efficiencies, where we are to calculate from plate to plate above the feed (see Fig. 3). Thus, say we know the states of the streams leaving plates $n + 1$ and n and specifically $\bar{y}_{n+1} = \bar{y}'_{n+1}$, x_n, x'_n, $\bar{y}'^{*}_n$, and wish to find the state at plate $n - 1$. From Eq. (22) we have

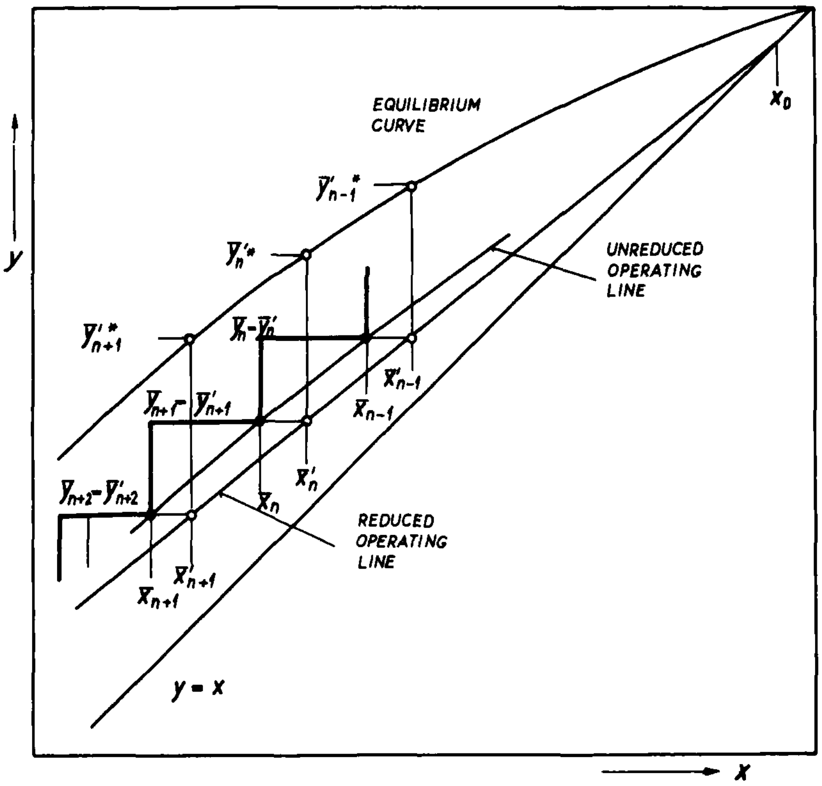

FIG. 3. Plate-to-plate construction with liquid carryover.

$$\bar{y}'_n = \bar{y}'_{n+1} + \bar{E}'_{MV}(\bar{y}'^{*}_n - \bar{y}'_{n+1}) \tag{31}$$

so we can draw the horizontal $\bar{y}'_n =$ constant and in particular find its intersection with the reduced operating line, giving $\bar{x}'_{n-1}$, from which we have immediately $\bar{y}'^{*}_{n-1}$, needed in the next step. If we are satisfied to know only the reduced concentrations, we may proceed immediately to find $\bar{y}'_{n-2}$ in the same manner. If, however, we also want to know the actual liquid concentrations, we can rearrange Eq. (12) to the form

$$(\dot{E}_n^{+''}/\dot{L}'_{n-1})\,(\bar{x}'_{n-1} - \bar{x}_n) = (1 + \dot{E}_n^{+''}/\dot{L}'_{n-1})(\bar{x}'_{n-1} - \bar{x}_{n-1}) \qquad (32)$$

from which we find $\bar{x}_{n-1}$. In this manner we can find the operating points on the reduced $(\bar{y}'_n, \bar{x}'_{n-1})$ and unreduced $(\bar{y}_n, \bar{x}_{n-1})$ operating lines. The feed point and condenser will require a special analysis, as they will usually have special carryovers. Other statements of the problem may require trial-and-error calculations.

Experimental values of the various constituent plate efficiencies and corresponding liquid carryovers discussed in this paper will be presented in a subsequent article of this series (XX).

List of Symbols

$\dot{C}$	molal vapor carryover (leakage) flow rate
$\dot{D}$	molal distillate flow rate
$\dot{E}$	molal liquid carryover (weeping) flow rate
E	plate efficiency
e'	relative liquid carryover [see Eq. (17)]
H	molal vapor enthalpy
h	molal liquid enthalpy
$\dot{L}$	molal liquid flow rate
m	slope of equilibrium curve [$= (dy/dx)^{*}$]
$\dot{Q}$	heat duty
R'	reflux ratio [see Eq. (16)]
$\dot{V}$	molal vapor flow rate
$\dot{W}$	molal residue flow rate
x	mole fraction of liquid constituent
y	mole fraction of vapor constituent
γ	weighting factor for vapor carryover [see Eq. (3)]
ϵ	weighting factor for liquid carryover [see Eq. (3)]

Indices

D	distillate
H	Hausen; vapor enthalpy
h	liquid enthalpy
i	*i*th constituent
L	liquid
M	Murphree
n	*n*th plate
V	vapor
W	residue
x	liquid constituent
y	vapor constituent
—	carryover in direction of flow of same phase
+	carryover in direction of flow of other phase
¯	mean value over stream cross section
″	corrected value
′	reduced value
*	equilibrium value

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